Lec-B

Simultaneous linear Equations

Gauss Elimination method

This is the elementary elimination method and it reduces the system of equations to an equivalent upper- ~~triangle~~ triangular system.

We consider the system as

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots a_{2n}x_n = b_2$$

$$a_{31}x_1 + a_{32}x_2 + \cdots \cdots a_{3n}x_n = b_3$$

$$\vdots$$

Step-I
To eliminate x_1 from the second equation, we multiply the first eq^n by $(-a_{21}/a_{11})$ and then adding the ~~require~~ obtained eq^n to the second equation.

Similarly, to eliminate x_1 from the third eq^n, we multiply the first eq^n by $(-a_{31}/a_{11})$ and then adding the obtained eq^n to the third eq^n

Step - II

Now we have to eliminate x_2 from the second and third equation

(1)

Gauss Elimination Method

Solve the following system of eqn using Gauss Elimination method

$$2x_1 + x_2 + x_3 = 10$$
$$3x_1 + 2x_2 + 3x_3 = 18 \quad \text{--- (1)}$$
$$x_1 + 4x_2 + 9x_3 = 16$$

Soln:-

First stage of elimination:

we first eliminate 'x_1' from the above system of equations. For this we first divide the eqn (1) by its pivot element (i.e. coefficient of x_1) we get

$$x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 = 5$$
$$3x_1 + 2x_2 + 3x_3 = 18$$
$$x_1 + 4x_2 + 9x_3 = 16.$$

To eliminate x_1 from the eqn (2) and eqn (3), multiply eqn (1) by 3 and then subtracting from the 2nd eqn.

$$\cancel{x_1 + \frac{1}{2}x_2 +}$$

$$3x_1 + 2x_2 + 3x_3 - 3x_1 - \frac{3}{2}x_2 - \frac{3}{2}x_3 = 18 - 15$$

$$= \frac{1}{2}x_2 + \frac{3}{2}x_3 = 3 \quad (a).$$

Subtract eqⁿ (1) from eqⁿ (3), as the coefficient of x_1 in 3rd eqⁿ is 1.

$$x_1 + 4x_2 + 9x_3 - x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 = 16 - 5$$

$$= \frac{7}{2}x_2 + \frac{17}{2}x_3 = 11 \quad (b)$$

Second stage of Elimination

In the second stage of elimination, we have two eqⁿ, i.e.

$$\left.\begin{aligned} \frac{1}{2}x_2 + \frac{3}{2}x_3 &= 3 \\ \frac{7}{2}x_2 + \frac{17}{2}x_3 &= 11 \end{aligned}\right\} \rightarrow (2)$$

Now, we have to eliminate x_2 from the above system of equations.

Divide the Ist eqⁿ of the equation by $\frac{1}{2}$ i.e. multiply by 2, we get

$$x_2 + 3x_3 = 6.$$

$$\frac{7}{2}x_2 + \frac{17}{2}x_3 = 11$$

multiply the eqⁿ (1) by $\frac{7}{2}$ and then subtract this equation from eqⁿ (2).

$$= \cancel{\frac{7}{2}x_2} + \frac{17}{2}x_3 - \cancel{\frac{7}{2}x_2} - \frac{21}{2}x_3 = 11 - 21$$

$$= -2x_3 = -10.$$

P.T.O

or $2x_3 = 10$

$x_3 = 5$

The backward Substitution

$$2x_1 + x_2 + x_3 = 10$$

$$\frac{1}{2}x_2 + \frac{3}{2}x_3 = 3$$

$$-2x_3 = -10$$

The last eq^n is

$$-2x_3 = -10$$

$$2x_3 = 10$$

$$x_3 = 5$$

~~Putt the value of $x_3 = 5$ in $\frac{1}{2}x_2 + \frac{3}{2}x_3 = 3$.~~

The second eq^n is

$$\frac{1}{2}x_2 + \frac{3}{2}x_3 = 3.$$

Putt the value of $x_3 = 5$ in above eq^n.

$$\frac{1}{2}x_2 + \frac{3}{2}\times 5 = 3$$

$$x_2 = -9.$$

The Ist eq^n is

$$2x_1 + x_2 + x_3 = 10$$

putting the values of x_2 and x_3 in the above equation.

$$\therefore \quad 2x_1 - 9 + 5 = 10$$

$$x_1 = 7.$$

Thus we obtained $x_1 = 7$, $x_2 = -9$ and $x_3 = 5$ by using Gauss Elimination method.

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Gauss Jordan Method

This is the modification of the Gauss Elimination method.
The method does not require back substitution to obtain the solution.

Step - I

Normalise the first eqn by dividing it by its pivot element (i.e. coefficient of x_1)

Step - II

Eliminate x_1 from the 2nd eqn and 3rd eqn either by subtracting or adding the 2nd eqn and 3rd eqn from the Ist equation.

Step - III

Normalise the 2nd equation by dividing it by its pivot element (i.e. coefficient of x_2).

Step - IV

Eliminate x_2 from the Ist equation and 3rd equation by subtracting eqn one and equation 3 third from the equation 2nd.

Step - V

Normalise the 3rd equation by dividing it by its pivot element (i.e. coefficient of x_3)

Step VI

Eliminate x_3 from the 1st eqn and 2nd 2nd equation

Ex-2 Solve the system

$$2x_1 + 4x_2 - 6x_3 = -8$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12$$

by using Gauss Jordan method.

Soln :-

Step - I

Normalise the 1st eqn by dividing it by its pivot elements (i.e. coefficient of x_1)

$$x_1 + 2x_2 - 3x_3 = -4$$
$$x_1 + 3x_2 + x_3 = 10$$
$$2x_1 - 4x_2 - 2x_3 = -12$$

Step - II

Eliminate x_1 from the 2nd equation and 3rd equation either by ~~divis~~ subtracting or adding the equations 1st and 2nd from the 1st equation.

$$x_1 + 2x_2 - 3x_3 = -4$$

$$0 + x_2 + 4x_3 = 14 \qquad [eq^n(2) - eq^n(1)]$$

$$0 - 8x_2 + 4x_3 = -4 \qquad [eq^n(3) - 2 \times eq^n(1)]$$

$$\Rightarrow \quad x_1 + 2x_2 - 3x_3 = -4$$

$$x_2 + 4x_3 = 14$$

$$-8x_2 + 4x_3 = -4$$

Step - III

Normalise the 2nd eq^n by dividing it by its pivot element (i.e. coefficient of x_2).

Note:- But the $eq^n(2)$ is already in normalised form..

So,

$$x_1 + 2x_2 - 3x_3 = -4$$

$$x_2 + 4x_3 = 14$$

$$-8x_2 + 4x_3 = -4.$$

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~~Step V~~

~~Normalise the 2nd 3rd equation by dividing it by its pivot element (i.e. coefficient of x_3).~~

Step VI

Eliminate x_3 from the 1st equation and 2nd equation.

Step - IV

Eliminate x_2 from the Ist equation and 3rd equation by subtracting or adding the ~~eqn (1)~~ eqn 1st and eqn third from the 2nd equation.

$$x_1 + 0 - 11x_3 = -32 \quad [eq^n(1) - 2 \times eq^n(2)]$$
$$x_2 + 4x_3 = 14$$
$$0 + 36x_3 = 108 \quad [eq^n(3) + 8 \times eq^n(2)]$$

$\therefore$

$$x_1 - 11x_3 = -32$$
$$x_2 + 4x_3 = 14$$
$$36x_3 = 108$$

Step - V

Normalise the third eqn by dividing it by its pivot element (i.e. coefficient of x_3).

$$x_1 - 11x_3 = -32$$
$$x_2 + 4x_3 = 14$$
$$x_3 = 3$$

step - VI

Eliminate x_3 from the eqn (1) and eqn (2)

$$x_1 + 0 = 1 \quad [eq^n (1) + 11 \times eq^n (3)]$$
$$x_2 + 0 = 2 \quad [eq^n (2) - 4 \times eq^n (3)]$$
$$x_3 = 3$$

$x_1 = 1$

$x_2 = 2$

$x_3 = 3$

Jacobi's Iteration method.

$$\begin{aligned}10x + y + z &= 12\\ x + 10y + 2z &= 13\\ 2x + 2y + 10z &= 14\end{aligned}\qquad \begin{aligned}2x+2y+10z &= 14\\ 10x+y+z &= 12\\ x+10y+2z &= 13\end{aligned}$$

Sol^n:-

The diagonal elements are not dominant in the given system of eq^{n}s.
Hence, let us re-arrange the eq^{n}s as follows -

$$\begin{aligned}10x + y + z &= 12\\ x + 10y + 2z &= 13.\\ 2x + 2y + 10z &= 14.\end{aligned}$$

Iteration - I

Let us take $x_0 = y_0 = z_0 = 0$, be the 1st approximation.

$$x_1 = \frac{1}{10}[12 - y_0 - z_0]$$

$$= \frac{1}{10}[12 - 0 - 0] = \frac{12}{10} = 6/5 = 1.2$$

$$y_1 = \frac{1}{10}[13 - x_0 - 2z_0]$$

$$= \frac{1}{10}[13 - 0 - 2\times 0] = 13/_{10} = 1.3.$$

$$z_1 = \frac{1}{10}[14 - 2x_0 - 2y_0]$$

$$= \frac{1}{10}[14 - 0 - 0] = 1.4$$

Date :
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Iteration No. 2

$$x_2 = \frac{1}{10}[12 - y_1 - z_1]$$

$$= \frac{1}{10}[12 - 1.3 - 1.4] = 0.93$$

$$y_2 = \frac{1}{10}[13 - x_1 - 2z_1]$$

$$= \frac{1}{10}[13 - 1.2 - 2 \times 1.4] = 0.9$$

$$z_2 = \frac{1}{10}[14 - 2x_1 - 2y_1]$$

$$= \frac{1}{10}[14 - 2 \times 1.2 - 2 \times 1.3] = 0.9$$

Iteration no. 3.

$$x_3 = \frac{1}{10}[12 - y_2 - z_2]$$

$$= \frac{1}{10}[12 - 0.9 - 0.9] =$$

$$y_3 = \frac{1}{10}[13 - x_2 - 2z_2]$$

$$= \frac{1}{10}[13 - 0.93 - 2 \times 0.9]$$

$$z_3 = \frac{1}{10}[14 - 2x_2 - 2y_2]$$

$$= \frac{1}{14}[14 - 2 \times 0.93 - 2 \times 0.9] =$$

Continue

upto Iteration No. 5.

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Then at the end of 5th iteration,

$x = 1.00182$

$y = 1.00749$

$z = 1.00298$

The exact values are -

$x = 1, \quad y = 1, \quad z = 1$